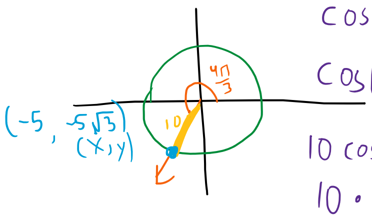
Warm-up!

Advanced Trig Day 5:
(3.13) Polar Coordinates

Homework: Polar Coordinates

Next Quiz
- Solving Trig Equations
- Solving Inverse Trig Equations
- Trig Identities
- Quiz is on Day 1 - Day 4

A) At what point does the terminal ray of $\theta = \frac{4\pi}{3}$ intersect a circle of radius 10?



$\cos\theta = \frac{x}{r}$ $\qquad \sin\theta = \frac{y}{r}$

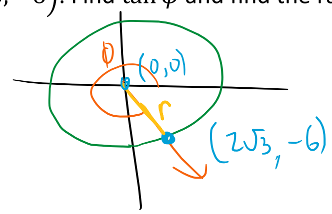
$\cos\left(\frac{4\pi}{3}\right) = \frac{x}{10}$ $\qquad \sin\left(\frac{4\pi}{3}\right) = \frac{y}{10}$

$10\cos\left(\frac{4\pi}{3}\right) = x$ $\qquad 10\sin\left(\frac{4\pi}{3}\right) = y$

$10 \cdot \left(-\frac{1}{2}\right) = x$ $\qquad 10 \cdot \left(-\frac{\sqrt{3}}{2}\right) = y$

-5 $\qquad -5\sqrt{3}$

B) The terminal ray of the angle ϕ passes through a circle at the point $\left(2\sqrt{3}, -6\right)$. Find $\tan\phi$ and find the radius of the circle.



$\tan\phi = \frac{y}{x} = \frac{-6}{2\sqrt{3}} = -\frac{3}{\sqrt{3}} = -\sqrt{3}$

$r = \sqrt{x^2+y^2}$

$r = \sqrt{(2\sqrt{3})^2 + (-6)^2}$

$r = \sqrt{4 \cdot 3 + 36}$

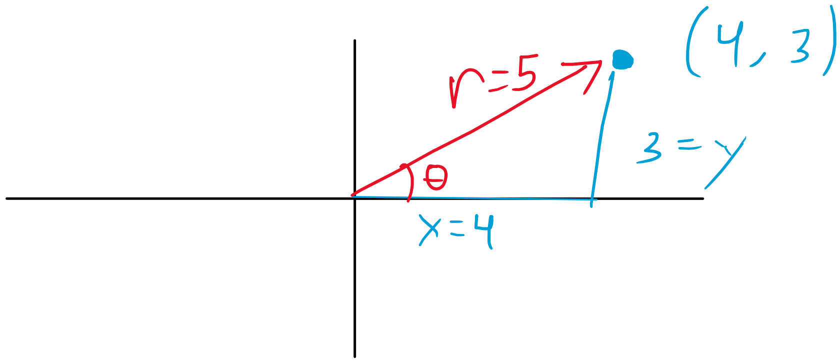
$r = \sqrt{12+36}$

$r = \sqrt{48}$

$r = \sqrt{16}\sqrt{3} = 4\sqrt{3}$

Polar Coordinates

Rectangular Coordinates: (x, y)



Polar Coordinates: (r, θ)

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{4^2 + 3^2}$$

$$r = \sqrt{16 + 9} = \sqrt{25} = 5$$

Polar Coordinate Example

r, θ

A.

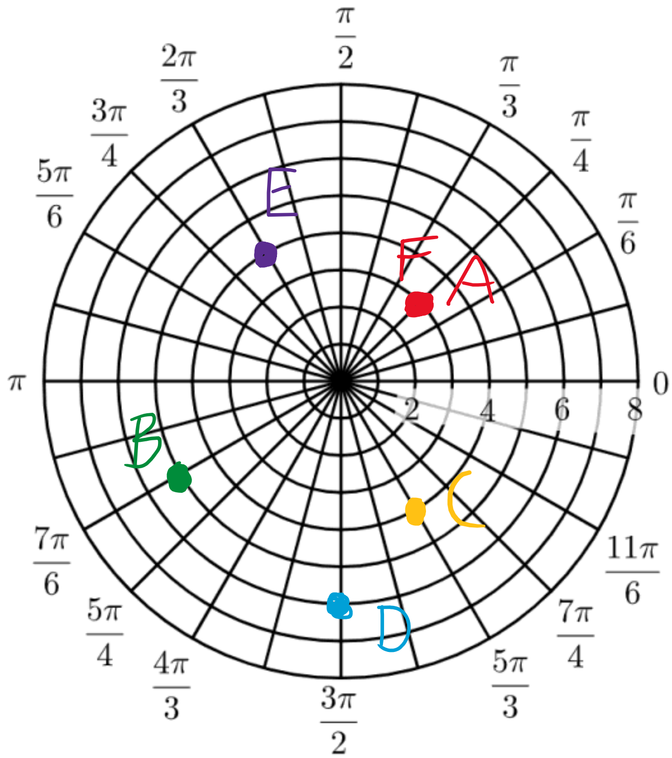
B.

C.

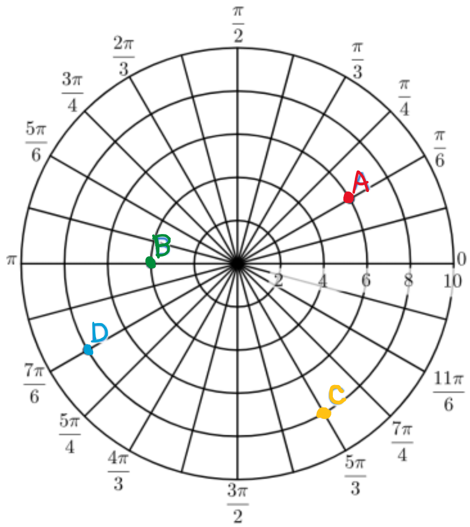
D.

E.

F.



Find three ways of labelling each point with polar coordinates



A $(6, \frac{\pi}{6})$ $(-6, \frac{7\pi}{6})$ $(6, -\frac{11\pi}{6})$ $(6, \frac{13\pi}{6})$

B $(4, \pi)$ $(-4, 0)$ $(4, -\pi)$ $(-4, 2\pi)$

C $(8, \frac{5\pi}{3})$ $(8, -\frac{\pi}{3})$ $(-8, \frac{2\pi}{3})$

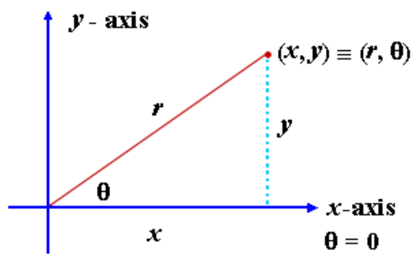
D $(8, \frac{7\pi}{6})$ $(-8, \frac{\pi}{6})$ $(8, -\frac{5\pi}{6})$

Which is not the same as the others?

$\left(7, \frac{4\pi}{3}\right)$ $\left(-7, \frac{\pi}{3}\right)$ $\left(-7, -\frac{4\pi}{3}\right)$ $\left(7, -\frac{2\pi}{3}\right)$

Converting Polar Coordinates to Rectangular

$(r, \theta) \longrightarrow (x, y)$



Since $\cos(\theta) = \frac{x}{r}$ ...

$r\cos\theta = x$

$x = r\cos\theta$

Since $\sin(\theta) = \frac{y}{r}$ ...

$r\sin\theta = y$

$y = r\sin\theta$

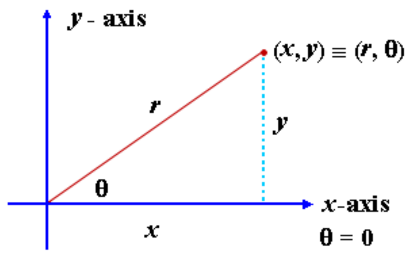
Convert from polar to rectangular

$$\left(5, -\frac{\pi}{3}\right)$$

$$\left(-4, \frac{3\pi}{4}\right)$$

$$\left(6, \frac{\pi}{2}\right)$$

Converting Rectangular Coordinates to Polar



$(x, y) \to (r, \theta)$

$r^2 = x^2 + y^2$

$r = \sqrt{x^2 + y^2}$

$\tan(\theta) = \frac{y}{x}$

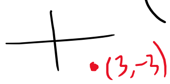
$\downarrow$

then solve for θ

pick correct quadrant

Converting from rectangular to polar

A. $\overset{x,\,y}{(3,-3)} \longrightarrow \overset{(r,\,\theta)}{\left(3\sqrt{2}, \frac{7\pi}{4}\right)}$



$r = \sqrt{x^2+y^2}$

$r = \sqrt{(3)^2+(-3)^2}$

$r = \sqrt{9+9}$

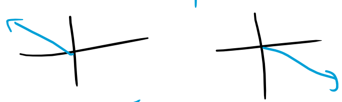
$r = \sqrt{18}$

$r = \sqrt{9}\sqrt{2} = 3\sqrt{2}$

$\tan\theta = y/x$

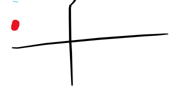
$\tan\theta = \frac{-3}{3}$

$\tan\theta = -1$



$\theta = \frac{3\pi}{4}$ $\qquad \theta = \frac{7\pi}{4}$

B. $\overset{x,\,y}{(-\sqrt{3}, 1)} \longrightarrow \overset{(r,\,\theta)}{\left(2, \frac{5\pi}{6}\right)}$



$r = \sqrt{x^2+y^2}$

$r = \sqrt{(-\sqrt{3})^2+(1)^2}$

$r = \sqrt{3+1}$

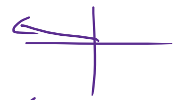
$r = \sqrt{4}$

$r = 2$

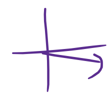
$\tan\theta = y/x$

$\tan\theta = \frac{1}{-\sqrt{3}}$

$\tan\theta = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$



$\theta = \frac{5\pi}{6}$



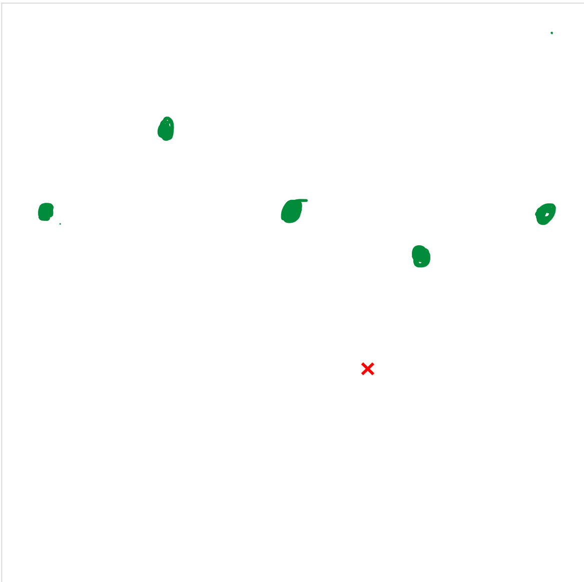
$\theta = \frac{11\pi}{6}$

Converting from rectangular to polar

C. $(0,10)$ D. $(-3,-4)$ E. $(-3,4)$

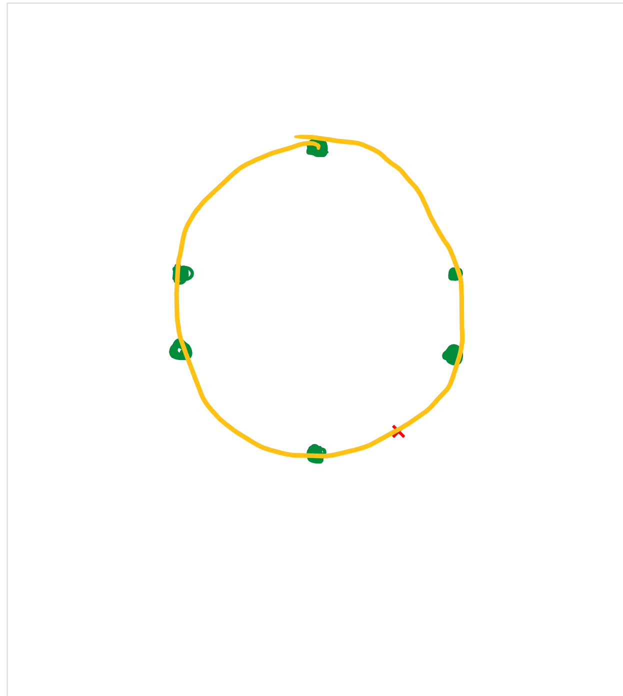
We are used to seeing the relationship between two variables illustrated with a rectangular graph. But we can also illustrate the relationship between two variables with a polar graph.

$$f(\theta) = \frac{12}{3 - \sin\theta}$$



0	4
$\pi/6$	4.8
$\pi/2$	6
$5\pi/6$	4.8
π	4
$3\pi/2$	3
2π	4

$$r = f(\theta)$$



We are used to seeing the relationship between two variables illustrated with a rectangular graph. But we can also illustrate the relationship between two variables with a polar graph.

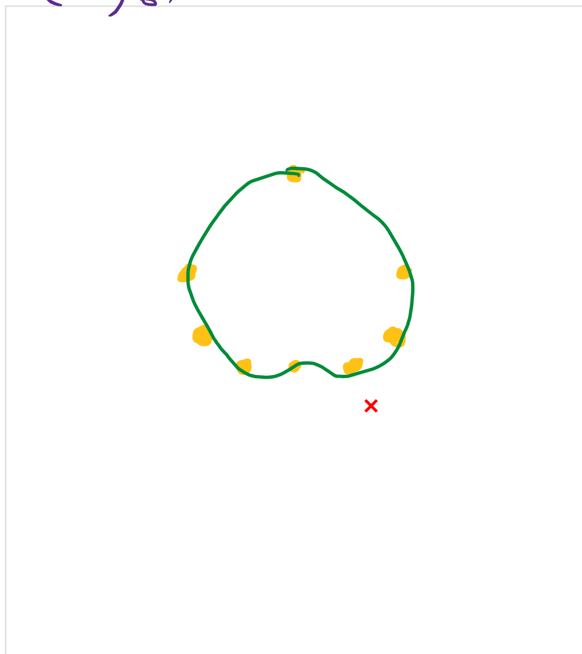
$$f(\theta) = -3 + 2\sin(\theta)$$

θ	r
0	-3
$\pi/6$	-2
$\pi/2$	-1
$5\pi/6$	-2
π	-3
$7\pi/6$	-4
$3\pi/2$	-5
$11\pi/6$	-4
2π	-3

$r \quad \theta$

$(-2, \frac{\pi}{6})$

$$r = f(\theta)$$



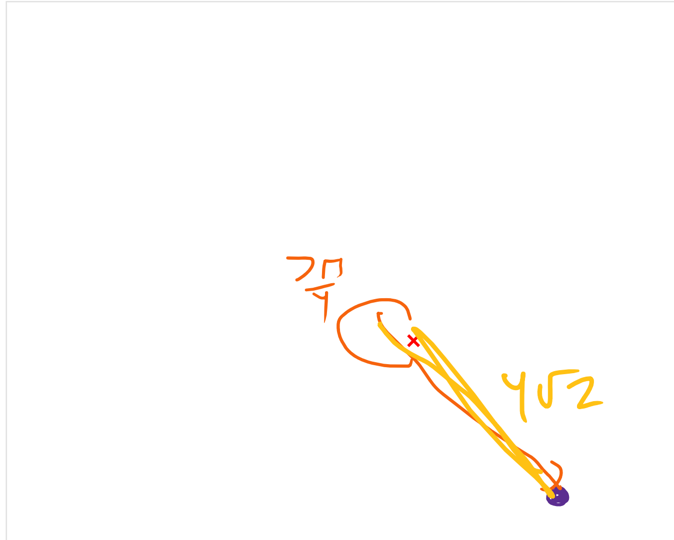
Sometimes we like to treat complex numbers as points in the "complex plane"

The number $4 - 4i$ is sort of like

$(4, -4)$

The point with rectangular coordinates $(4, -4)$

AKA the point with polar coordinates $(\overset{r}{4\sqrt{2}}, \overset{\theta}{\frac{7\pi}{4}})$



Real part of the number is always the x-value of the point

Imaginary part of the number is always the y-value of the point

Let $z = 6 + 2\sqrt{3}i$

Find the rectangular coordinates of the point that corresponds with the value of z

Find the polar coordinates of the point that corresponds with the value of z

Let $z = 6 + 2\sqrt{3}i$

Rather than write z as $\left(6, 2\sqrt{3}\right)$ or as $\left(4\sqrt{3}, \frac{\pi}{6}\right)$, we sometimes write it as

$$\left(4\sqrt{3}\cos\left(\frac{\pi}{6}\right)\right) + i\left(4\sqrt{3}\sin\left(\frac{\pi}{6}\right)\right)$$

Rectangular

Polar

Let's call this the "polar form" of the complex number.

What's nice about this is that it's easy to quickly see what the rectangular coordinates or the polar coordinates of the point are.

If the point corresponding with a complex number has polar coordinates (r, θ), then we write the polar form of the complex number as

$$(r\cos\theta) + i(r\sin\theta)$$

$4 - 4i$

$\downarrow$

rectangular $(4, -4)$

$\downarrow$

Polar $\left(4\sqrt{2}, \frac{7\pi}{4}\right)$

$$\left(4\sqrt{2}\cos\frac{7\pi}{4}\right) + i\left(4\sqrt{2}\sin\frac{7\pi}{4}\right)$$

$$4\sqrt{2}\cdot\left(\frac{\sqrt{2}}{2}\right) + i\left(4\sqrt{2}\left(-\frac{\sqrt{2}}{2}\right)\right)$$

$$4 - 4i$$